AP Calculus AB — Homework 0213 — Name:

Dr. Paul L. Bailey — Thursday, February 13, 2025

Write your homework *neatly*, *in pencil*, on blank white $8\frac{1}{2} \times 11$ printer paper. Always *write the problem*, or at least enough of it so that your work is readable. When appropriate, *write in sentences*.

The main theory of section 4.2 is summarized below.

Theorem 1. (Rolle's Theorem)
Let f be continuous on a closed interval $[a,b]$ and differentiable on (a,b). Suppose that $f(a) = f(b) = 0$. Then there exists $c \in (a,b)$ such that $f'(c) = 0$.

Theorem 2. (Mean Value Theorem (MVT))
Let f be continuous on a closed interval $[a,b]$ and differentiable on (a,b). Then there exists $c \in (a,b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Corollary 1. *Let f be continuous on a closed interval $[a,b]$ and differentiable on (a,b). If $f'(x) = 0$ for all $x \in (a,b)$, then $f(x) = f(a) = f(b)$ for all $x \in [a,b]$.*

Corollary 2. *Let f and g be continuous on a closed interval $[a,b]$ and differentiable on (a,b). Suppose $f'(x) = g'(x)$ for all $x \in (a,b)$. Then there exists $C \in \mathbb{R}$ such that $g(x) = f(x) + C$ for all $x \in [a,b]$.*

Problem 1 (Thomas §4.2 # 23)**.** Suppose that $f(1) = 3$ and that $f'(x) = 0$ for all $x \in (0,2)$. Must $f(x) = 3$ for all $x \in (0,2)$? for all $x \in [0,2]$? Give reasons for your answer.

Problem 2 (Thomas §4.2 # 24)**.** Suppose that $f(0) = 5$ and that $f'(x) = 2$ for all $x \in (-2,2)$. Must $f(x) = 2x + 5$ for all $x \in (-2,2)$? Give reasons for your answer.

Problem 3 (Thomas §4.2 # 27)**.** Find all possible functions with the given derivative.

- **(a)** x
- **(b)** x^2
- **(c)** x^3

Problem 4 (Thomas §4.2 # 28)**.** Find all possible functions with the given derivative.

- **(a)** $2x$
- **(b)** $2x - 1$
- **(c)** $3x^2 + 2x - 1$

Problem 5 (Thomas §4.2 # 41)**.** A body moves with acceleration $a = d^2s/dt^2$, initial velocity $v(0)$, and initial position $s(0)$ along a coordinate line, where

$$a = 32, v_0 = 20, s_0 = 5$$

Find the body's position at time t.

Problem 6. Compute dy/dx. Simplify.

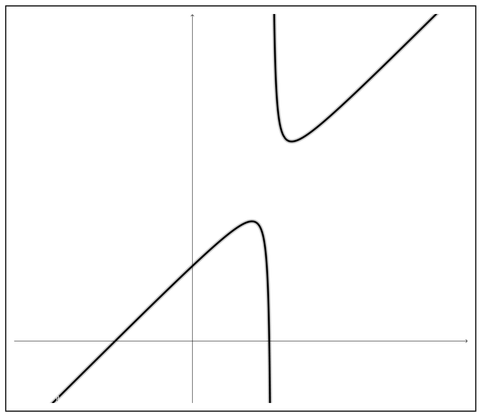
(a) $y = \dfrac{x^2 - 4}{x - 2}$

(b) $y = \dfrac{x^2 + 3x - 1}{x - 2}$

(c) $y = \sec^2 x - \tan^2 x$

Problem 7. Let

$$f(x) = \frac{x^2 - 15}{x - 4}.$$



(a) Solve $f'(x) = 0$.

(b) Find the domain and range of f.

(c) The graph of f has two linear asymptotes. Write the equations for these lines.

Problem 8. Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = \sin(1 + x^2)$. Find all $x \in \mathbb{R}$ such that f is differentiable at x.

Problem 9 (Thomas Ch 2 Practice # 15)**.** Find

$$\lim_{x \to 0} \frac{\frac{1}{2+x} - \frac{1}{x}}{x}.$$

Problem 10 (Thomas §2.6 # 47)**.** Show that the equation

$$x^3 - 15x + 1 = 0$$

has three solutions in the interval $[-4, 4]$. (Hint: use IVT.)